Research project ranking

Rudolf Scitovski[1]
Department of Mathematics, University of Osijek
Trg Ljudevita Gaja 6, HR – 31 000 Osijek, Croatia
e-mail: scitowsk@mathos.hr

Mario Vinković
Faculty of Law, University of Osijek
Stjepana Radića 13, HR – 31 000 Osijek, Croatia
e-mail: mvinkovi@pravos.hr

Kristian Sabo
Department of Mathematics, University of Osijek
Trg Ljudevita Gaja 6, HR – 31 000 Osijek, Croatia
e-mail: ksabo@mathos.hr

Ana Kozić
Faculty of Education, University of Osijek
Trg Sv. Trojstva 3, HR – 31 000 Osijek, Croatia
e-mail: akozic@unios.hr

Abstract. The paper discusses the problem of ranking research projects based on the assessment obtained from two or several independent reviewers. Each reviewer assesses several project features, and the total score is defined as the weighted arithmetic mean, where the weights of features are determined according to the well-known AHP method. In this way, it is possible to identify each project by a point in n-dimensional space. The ranking is performed on the basis of the distance of each project to the perfectly assessed project. Thereby the application of different metric functions is analyzed. We believe it is inappropriate to use a larger number of decimal places if two projects are almost equidistant (according to some distance function) to the perfectly assessed project. In that case, it would be more appropriate to give priority to the project with more balanced assessments obtained from different reviewers, which is achieved by combining different distance functions. The method is illustrated by several simple examples and applied by ranking internal research projects at Josip Juraj Strossmayer University of Osijek.

Keywords: Multi-criteria decision making; Project evaluation; ℓ_p-distance; AHP

1 Introduction

The problem of ranking research projects (see e.g. Collan et al. (2013); Mandic et al. (2014); Mardani et al. (2015); Ž. Turkalj et al. (2016)) as well as ranking departments, institutes and universities (see e.g. Daraio et al. (2015); Kadziński and Słowiński (2015); Rad et al. (2011)) has long been present in the scientific literature. Most approaches

[1]Corresponding author: Rudolf Scitovski, e-mail: scitowsk@mathos.hr, telephone number: ++385-31-224-800, fax number: ++385-31-224-801

use different multi-criteria decision-making methods, but mostly the Analytic Hierarchy Process (AHP) (see e.g. (Saaty, 1980, 1990)). In the paper (Ž. Turkalj et al., 2016), the AHP method and adaptive Mahalanobis clustering are combined (Morales-Esteban et al., 2014).

This paper is organized as follows. The problem is stated and one practical problem of ranking internal research projects at the University of Osijek is discussed in Section 2. The definition of ordering on a set of projects in terms of various distance functions is also introduced. An example indicating the basic problems that might occur is constructed. In Section 3, an analysis of various distance functions, i.e. the Manhattan d_1-distance, the Euclidean d_2-distance, and the Chebyshev d_∞-distance, is performed. The situation when two or more projects are evaluated differently by various reviewers, and yet roughly equally ranked by using some distance function, is especially considered. In that case, we believe priority should be given to projects with more balanced assessments, which can be achieved by combining different distance functions.

The real ranking problem of internal research projects at the University of Osijek is described in Section 4, and finally, some conclusions are given in Section 5.

2 Problem statement

Let $\mathcal{P} = \{\pi^{(1)}, \ldots, \pi^{(m)}\}$ be a set of projects. Suppose that each project is assessed by $n \geq 1$ independent reviewers based on the review form in which $k \geq 1$ features $f_1, \ldots, f_k$ (e.g. the quality and relevance of a research proposal, the quality of applicants, etc.; see Example 1) are assessed. The corresponding weight $w_j > 0$ will be associated to each of k features f_j which will be assessed. This can be done by using the AHP method (see Saaty (1980, 1990)). Without loss of generality, let us suppose that $\sum\limits_{s=1}^{k} w_s = 1$. By $r_{j1}^{(i)}, \ldots, r_{jk}^{(i)} \in [1,5]$ we denote grades of features $f_1, \ldots, f_k$ given for the project $\pi^{(i)} \in \mathcal{P}$ by the j-th reviewer.

Furthermore, let

$$r_j^{(i)} = \sum_{s=1}^{k} w_s r_{js}^{(i)}, \quad j = 1, \ldots, n, \quad i = 1, \ldots, m, \tag{1}$$

be the average weighted grade (AWG) of the project $\pi^{(i)}$ obtained from the j-th reviewer. In this way, we are able to associate a vector (point)

$$a^{(i)} = (r_1^{(i)}, \ldots, r_n^{(i)}) \in \mathbb{R}^n, \quad i = 1, \ldots, m, \tag{2}$$

from n-dimensional vector space $\mathbb{R}^n$ to each project $\pi^{(i)} \in \mathcal{P}$. So we have established a bijection between the set of all projects $\mathcal{P}$ and the set $\mathcal{A} = \{a^{(i)} \in \mathbb{R}^n \colon i = 1, \ldots, m\}$ of points in the space $\mathbb{R}^n$.

Example 1. *In 2015, the University of Osijek announced an internal call for proposals for research projects INGI-2015*[2] *to encourage cooperation between its researchers and prominent researchers from other (especially foreign) universities. 30 candidates from the STEM area and 10 Social Sciences and Humanities candidates submitted their applications to the call. The evaluation was carried out based on reviews by independent reviewers, one of whom is affiliated with the field of the research proposal in question and the other comes from a different, but related field. Reviewers evaluated features* $f_1, \dots, f_6$ *(given in Table 1) with grades from the interval* $[1,5]$*. The Committee for Research Project Evaluation defined weights* $w_1, \dots, w_6 > 0$ *of particular features by using the AHP method (see also Table 1). In that way, for each of* $m = 40$ *projects* $\pi^{(i)}$ *the corresponding vector* $a^{(i)} \in \mathbb{R}^2$ *is uniquely determined, whose components are AWGs of all features of the first and the second reviewer*

$$a^{(i)} = (r_1^{(i)}, r_2^{(i)}) \in \mathbb{R}^2, \quad i = 1, \dots, m. \tag{3}$$

	Features	Weights w_i
f_1:	The quality and relevance of the research proposal	0.25
f_2:	The quality of applicants	0.15
f_3:	The quality of guest researchers	0.35
f_4:	Research feasibility study	0.10
f_5:	Financial plan	0.10
f_6:	Inclusion of students	0.05

Table 1: Elements assessed by reviewers from Example 1 with corresponding weights

Furthermore, by $\pi^\star$ we will denote a *perfectly assessed project* to which the point $a^\star = (5, \dots, 5) \in \mathbb{R}^n$ is associated in space $\mathbb{R}^n$. The project $\pi^{(i)}$ is considered to be ranked better than the project $\pi^{(j)}$ if the point $a^{(i)}$ is closer to the point $a^\star$ in terms of some distance function. In this sense, we introduce the following definition.

Definition 1. Let $\mathcal{P} = \{\pi^{(1)}, \dots, \pi^{(m)}\}$ be a set of projects, $\mathcal{A} \subset \mathbb{R}^n$ a set of corresponding points defined by (2), $\pi^\star$ the perfectly assessed project to which we associate the point $a^\star = (5, \dots, 5) \in \mathbb{R}^n$, and let $d\colon \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}_+$ be some distance function. The project $\pi^{(i)}$ is said to be better d-ranked than the project $\pi^{(j)}$ and we write $\pi^{(i)} \overset{(d)}{\succeq} \pi^{(j)}$ if and only if there holds $d(a^{(i)}, a^\star) \le d(a^{(j)}, a^\star)$, i.e.,

$$\pi^{(i)} \overset{(d)}{\succeq} \pi^{(j)} \Leftrightarrow d(a^{(i)}, a^\star) \le d(a^{(j)}, a^\star).$$

Furthermore, we say that a set of projects $\mathcal{P}$ is d-ranked if $\pi^{(1)} \overset{(d)}{\succeq} \cdots \overset{(d)}{\succeq} \pi^{(j)} \overset{(d)}{\succeq} \cdots \overset{(d)}{\succeq} \pi^{(m)}$ and j is a d-rank of the project $\pi^{(j)}$.

[2]See: `http://www.unios.hr/ingi2015/`

Let $K_r^{(d)} = \{x \in \mathbb{R}^n \colon d(x, a^\star) \le r\}$ be a hyperball of radius $r > 0$ with the center in the point $a^\star$ in metric space $\mathbb{R}^n$ with distance function $d \colon \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}_+$. Obviously, $\pi^{(i)}$ is strongly better d-ranked than $\pi^{(j)}$ ($\pi^{(i)} \overset{(d)}{\succ} \pi^{(j)}$) if the point $a^{(i)}$ is situated in hyperball $K_r^{(d)}$ of smaller radius. Projects $\pi^{(i)}$ and $\pi^{(j)}$ are equally d-ranked if the corresponding points $a^{(i)}$ and $a^{(j)}$ lie in the same hypercircle $\partial K_r^{(d)}$. In this way, we introduce complete ordering on the set of points $\mathcal{A}$ and a unique ranking list of projects $\mathcal{P}$.

The proposed method of project ranking allows the application of various distance functions, and in this paper we will particularly analyze the application of the Manhattan d_1-distance function, the Euclidean d_2-distance function, and the Chebyshev d_∞-distance function.

Remark 1. Note that two projects $\pi^{(i)}$, $\pi^{(j)} \in \mathcal{P}$ (see Fig. 1)

- have the same d_1-rank if $d_1(a^{(i)}, a^\star) = d_1(a^{(j)}, a^\star)$, i.e., if the arithmetic means of their grades are equal: $\frac{1}{n} \sum\limits_{s=1}^{n} r_s^{(i)} = \frac{1}{n} \sum\limits_{s=1}^{n} r_s^{(j)}$;
- have the same d_2-rank if $d_2(a^{(i)}, a^\star) = d_2(a^{(j)}, a^\star)$;
- have the same d_∞-rank if $d_\infty(a^{(i)}, a^\star) = d_\infty(a^{(j)}, a^\star)$, i.e., if the highest grades obtained for some feature are equal: $\max\limits_{s=1,\dots,n} r_s^{(i)} = \max\limits_{s=1,\dots,n} r_s^{(j)}$.

Example 2. *Let $m = 7$ and $n = 2$. Average grades awarded to projects by two independent reviewers are given in Table 2. In this way, the set $\mathcal{A} = \{a^{(i)} = (x_i, y_i) \in \mathbb{R}^2 \colon i = 1, \dots, 7\}$ of the corresponding points is determined. The table also gives distances of each project to the perfectly assessed project $\pi^\star$ by using d_1, d_2 and d_∞ distance functions. In addition to the set of points $\mathcal{A}$, a few d_1-circles suggesting a d_1-rank of projects are shown in Fig. 1a. Similarly, Fig. 1b and Fig. 1c contain a few d_2-circles and a few d_∞-circles suggesting a d_2-rank of projects and a d_∞-rank of projects, respectively.*

Project	$\pi^{(1)}$	$\pi^{(2)}$	$\pi^{(3)}$	$\pi^{(4)}$	$\pi^{(5)}$	$\pi^{(6)}$	$\pi^{(7)}$
`Rev#1` (x_i)	3.0	2.0	3.4	4.1	4.2	4.9	4.4
`Rev#2` (y_i)	2.8	4.5	4.1	3.4	4.6	3.9	4.4
$d_1(\pi^{(i)}, a^\star)$	4.2	3.5	2.5	2.5	1.2	1.2	1.2
$d_2(\pi^{(i)}, a^\star)$	3.0	3.0	1.8	1.8	0.9	1.1	0.8
$d_\infty(\pi^{(i)}, a^\star)$	2.2	3.0	1.6	1.6	0.8	1.1	0.6

Table 2: Project grades and distances to the perfectly assessed project $\pi^\star$

Table 3 gives d_1, d_2, and d_∞ ranking lists of projects from Example 2. Note that projects $\pi^{(3)}, \pi^{(4)}$, i.e. projects $\pi^{(5)}, \pi^{(6)}, \pi^{(7)}$, lie in the same d_1-circle and have the same d_1-rank. Similarly, projects $\pi^{(1)}, \pi^{(2)}$, i.e. projects $\pi^{(3)}, \pi^{(4)}$, lie in the same d_2-circle and have the same d_2-rank. A similar problem also occurs in the application of the d_∞-distance. These problems will be analyzed in detail in the next section.

(a) Manhattan d_1-distance (b) Euclidean d_2-distance (c) Chebyshev d_∞-distance

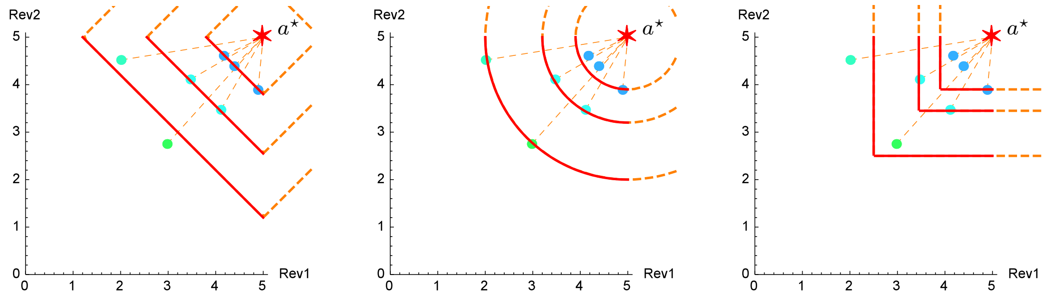

Figure 1: Distances to the perfectly assessed project $\pi^\star$ represented by the point $a^\star = (5, 5)$

Rank	Manhattan d_1-distance	Euclidean d_2-distance	Chebyshev d_∞-distance
1	$\pi^{(5)}, \pi^{(6)}, \pi^{(7)}$	$\pi^{(7)}$	$\pi^{(7)}$
2	$\pi^{(3)}, \pi^{(4)}$	$\pi^{(5)}$	$\pi^{(5)}$
3	$\pi^{(2)}$	$\pi^{(6)}$	$\pi^{(6)}$
4	$\pi^{(1)}$	$\pi^{(3)}, \pi^{(4)}$	$\pi^{(3)}, \pi^{(4)}$
5	–	$\pi^{(1)}, \pi^{(2)}$	$\pi^{(1)}$
6	–	–	$\pi^{(2)}$
7	–	–	–

Table 3: Ranking of projects from Example 2 by using various distance functions

3 Comparison of the application of various metric functions

As already mentioned in the previous section, two projects will be equally d_1-ranked if the arithmetic means of their grades (2) are equal. It is immediately clear that if the Manhattan d_1-distance function is applied, the rank of some project $\pi \in \mathcal{P}$ will be influenced only by arithmetic means of grades (2), and diversity of individual grades (2) awarded by various reviewers will not affect the d_1-rank of the project at all.

Unlike the d_1-rank, the d_2-rank and the d_∞-rank will depend on grade dispersion (2) referring to the project under consideration.

As an illustration, let us consider two projects represented by the points a and a^0 from the plane $\mathbb{R}^2$ (see Fig. 2), which are equally d_1-ranked, i.e., they equally differ from $a^\star$ by the Manhattan distance: $d_1(a^0, a^\star) = d_1(a, a^\star) =: r$. In Fig. 2, it can be seen that the point $a^0 \in \mathbb{R}^2$ represents the project $\pi^0 \in \mathcal{P}$, whose AWGs obtained from both reviewers are mutually equal. Among all projects $\pi \in \mathcal{P}$ for which $d_1(a, a^\star) = r$, the project π^0 attains the best d_2-rank (see Fig. 2a) and the best d_∞-rank (see Fig. 2b).

This means that the application of d_2 and d_∞ distances prefers uniform evaluation grades, unlike the Manhattan distance that takes into consideration only the arithmetic means of AWGs obtained from all reviewers. Practically, in case we have projects that are evaluated similarly and we want to give priority to the project with more uniform

(a) $\pi^0(a^0)$ attains the best d_2-rank (b) $\pi^0(a^0)$ attains the best d_∞-rank

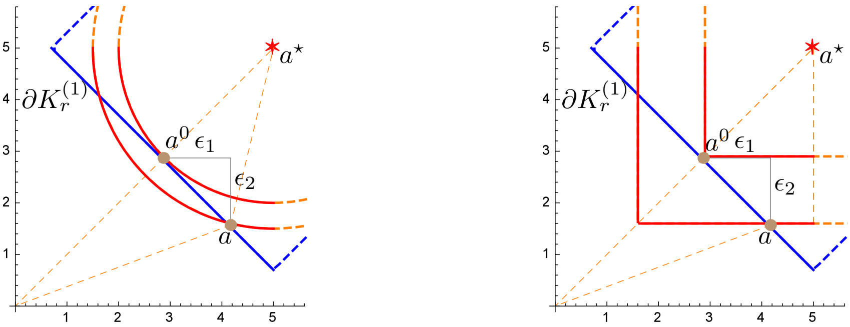

Figure 2: d_p, $p \geq 2$ distances prefer uniform evaluation grades

evaluation grades, we should use either the d_2 or the d_∞ distance, and if we do not want to give priority to such project, we should use the Manhattan distance.

A generalized principle for the case of $n > 1$ reviewers is described in the following theorem.

Theorem 1. *Let $n > 1$ and let $\partial K_r^{(1)} = \{x \in \mathbb{R}_+^n : d_1(x, a^\star) = r,\ r > 0\}$ be part of the Manhattan hypercircle of radius $r > 0$ with the center at the point $a^\star$. Then the shortest d_p, $p \in \{2, \infty\}$, distance from the point $a^\star$ to the hypercircle $\partial K_r^{(1)}$ is attained at the point $a^0 = (r, \ldots, r) \in \partial K_r^{(1)}$, i.e.,*

$$d_p(\partial K_r^{(1)}, a^\star) = \min_{a \in \partial K_r^{(1)}} d_p(a, a^\star) = d_p(a^0, a^\star), \quad p \in \{2, \infty\}. \tag{4}$$

Proof. First, let us note that for all $a \in \partial K_r^{(1)}$ there is $\epsilon \in \mathbb{R}^n$, such that

$$a = a^0 + \epsilon = (r + \epsilon_1, \ldots, r + \epsilon_n), \quad \text{where} \quad \sum_{i=1}^{n} \epsilon_i = 0. \tag{5}$$

In order to prove the assertion for $p = 2$, let us suppose that $r \in \mathbb{R}_+$ is fixed, define the function

$$\varphi \colon \mathbb{R}^n \to \mathbb{R}_+, \quad \varphi(\epsilon) = d_2^2(a, a^\star) = (5 - r - \epsilon_1)^2 + \cdots + (5 - r - \epsilon_n)^2,$$

and consider the following constrained optimization problem:

$$\min_{\{(\epsilon_1, \ldots, \epsilon_n) \in \mathbb{R}^n : \sum_{i=1}^n \epsilon_i = 0\}} \varphi(\epsilon_1, \ldots, \epsilon_n). \tag{6}$$

The corresponding Lagrange function for problem (6) is

$$L(\epsilon_1, \ldots, \epsilon_n, \lambda) = (5 - r - \epsilon_1)^2 + \cdots + (5 - r - \epsilon_n)^2 + \lambda \sum_{i=1}^{n} \epsilon_i.$$

From $\frac{\partial L(\epsilon_1, \ldots, \epsilon_n)}{\partial \epsilon_i} = \lambda - 2(5 - \epsilon_i - r) = 0$, we obtain $\epsilon_i = \frac{1}{2}(10 - \lambda - 2r)$, $i = 1, \ldots, n$. Finally, because $\sum_{i=1}^n \epsilon_i = 0$, we obtain $\lambda = 10 - 2r$, i.e. $\epsilon_i = 0$, $i = 1, \ldots, n$. Since the function φ is a strongly convex (quadratic) function, the assertion is proved.

In order to prove the assertion for $p=\infty$, let us define the function

$$\psi\colon \mathbb{R}^n \to \mathbb{R}, \quad \psi(\varepsilon) = d_\infty(a_\varepsilon, a^\star) = \max\{|5-r-\varepsilon_1|, \dots, |5-r-\varepsilon_n|\}$$

and consider the following constrained optimization problem:

$$\min_{\{(\varepsilon_1,\dots,\varepsilon_n)\in\mathbb{R}^n\colon \sum_{i=1}^n \varepsilon_i = 0\}} \psi(\varepsilon_1,\dots,\varepsilon_n), \tag{7}$$

Let $z = \max\{|5-r-\varepsilon_1|, \dots, |5-r-\varepsilon_n|\}$. Problem (7) is reduced to a linear programming problem:

$$\begin{aligned} & z \to \min \\ s.t. \quad & \sum_{i=1}^n \varepsilon_i = 0, && (8) \\ & 5-r-\varepsilon_i \le z, \quad i=1,\dots,n, && (9) \\ & -5+r+\varepsilon_i \le z, \quad i=1,\dots,n, && (10) \\ & \varepsilon_i \in \mathbb{R}_+. && (11) \end{aligned}$$

This problem can be solved explicitly. By summing conditions (9) and using (8) we get $5-r \le z$. Analogously, by summing conditions (10) and using (8) we obtain $-5+r \le z$, and finally, $|5-r| = \max\{5-r, -5+r\} \le z$. Since z can be minimal, it is obvious that the optimal $z^\star = |5-r| = \psi(0,\dots,0)$. □

Let us now consider the set of projects $\mathcal{P}^0 \subseteq \mathcal{P}$ which are equally d_2-ranked. As can be seen in Fig. 3a, the Chebyshev d_∞ distance project $\pi^0 \in \mathcal{P}^0$ with uniform evaluation grades $a^0 = (r,r)$ is recognized as best since $d_\infty(a^0, a^\star) \le d_\infty(a, a^\star)$, for all $a \in \mathcal{P}^0$. At the same time, the project $\pi^0 \in \mathcal{P}^0$ is d_1-ranked worst, and the corresponding vector a^0 has the smallest ℓ_2-norm (see Fig. 3b).

(a) $d_\infty(a^0, a^\star) < d_\infty(a, a^\star)$ (b) $\|a^{(i)}\|_p = \|a^{(j)}\|_p$, $p=1,2$

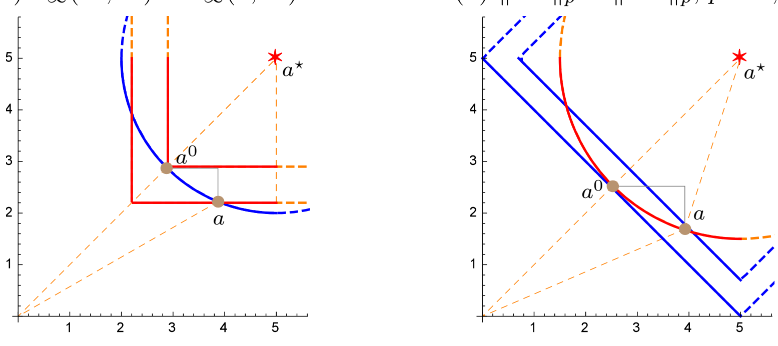

Figure 3: Two projects equally d_2-ranked

The following theorem gives a generalization of the aforementioned claims for $n>1$ and shows that among all projects $\pi \in \mathcal{P}^0$, the highest d_∞-rank is attributed to the project $\pi^0 \in \mathcal{P}^0$ with uniform evaluation grades $a^0 = (r,\dots,r)$. At the same time, the project $\pi^0 \in \mathcal{P}^0$ has the lowest d_1-rank, and the corresponding vector a^0 has the smallest ℓ_2-norm.

Theorem 2. *Let $n > 1$ and let $\partial K_r^{(2)} = \{x \in \mathbb{R}_+^n \colon d_2(x, a^\star) = r,\ r > 0\}$ be part of the Euclidean hypercircle of radius $r > 0$ with the center in the point $a^\star$.*

(i) The shortest d_∞ distance from the point $a^\star$ to the hypercircle $\partial K_r^{(2)}$ is attained at the point $a^0 = (r, \ldots, r) \in \partial K_r^{(2)}$, i.e.,

$$d_\infty(\partial K_r^{(2)}, a^\star) = \min_{a \in \partial K_r^{(2)}} d_\infty(a, a^\star) = d_\infty(a^0, a^\star). \tag{12}$$

(ii) For all pairs $a^{(i)}, a^{(j)} \in \partial K_r^{(2)}$, there holds

$$d_1(a^{(i)}, a^\star) \geq d_1(a^{(j)}, a^\star) \quad \Leftrightarrow \quad \|a^{(i)}\|_2 \leq \|a^{(j)}\|_2, \tag{13}$$

and particularly, the greatest d_1-distance from the point $a^\star$ to the hypercircle $\partial K_r^{(2)}$ is attained at the point $a^0 = (r, \ldots, r) \in \partial K_r^{(2)}$, i.e.,

$$d_1(\partial K_r^{(2)}, a^\star) = \max_{a \in \partial K_r^{(2)}} d_1(a, a^\star) = d_1(a^0, a^\star). \tag{14}$$

Proof. First, let us note that for all $a \in \partial K_r^{(2)}$ there exists $\epsilon \in \mathbb{R}^n$, such that

$$a = a^0 + \epsilon = (r + \epsilon_1, \ldots, r + \epsilon_n), \quad \text{where} \quad \sum_{i=1}^n \epsilon_i \leq 0. \tag{15}$$

Namely, the points $a, a^0 \in \partial K_r^{(2)}$ have the same distance from the point $a^\star$, and there holds

$$\begin{aligned}
&(5 - r - \epsilon_1)^2 + \cdots + (5 - r - \epsilon_n)^2 = (5 - r)^2 + \cdots + (5 - r)^2 \\
\Rightarrow\ & \epsilon_1^2 + \cdots + \epsilon_n^2 + 2(5 - r)(\epsilon_1 + \cdots + \epsilon_n) = 0 \\
\Rightarrow\ & \epsilon_1 + \cdots + \epsilon_n \leq 0.
\end{aligned}$$

Let us define the function

$$\psi \colon \mathbb{R}^n \to \mathbb{R}, \quad \psi(\varepsilon) = d_\infty(a_\varepsilon, a^\star) = \max\{|5 - r - \varepsilon_1|, \ldots, |5 - r - \varepsilon_n|\}$$

and consider the following constrained optimization problem

$$\min_{\{(\varepsilon_1, \ldots, \varepsilon_n) \in \mathbb{R}^n \colon \sum_{i=1}^n \varepsilon_i \leq 0\}} \psi(\varepsilon_1, \ldots, \varepsilon_n). \tag{16}$$

Let $z = \max\{|5 - r - \varepsilon_1|, \ldots, |5 - r - \varepsilon_n|\}$. Problem (16) is reduced to a linear programming problem

$$\begin{aligned}
& z \to \min \\
s.t.\quad & \sum_{i=1}^n \varepsilon_i \leq 0, && (17) \\
& 5 - r - \varepsilon_i \leq z, \quad i = 1, \ldots, n, && (18) \\
& -5 + r + \varepsilon_i \leq z, \quad i = 1, \ldots, n, && (19) \\
& \varepsilon_i \in \mathbb{R}. && (20)
\end{aligned}$$

This problem can be solved explicitly. By summing conditions (18) we get

$$n(5-r)-\sum_{i=1}^{n}\varepsilon_i \le nz,$$

by summing conditions (19) we get

$$n(-5+r)+\sum_{i=1}^{n}\varepsilon_i \le nz,$$

and finally,

$$\max\{n(5-r)-\sum_{i=1}^{n}\varepsilon_i, n(-5+r)+\sum_{i=1}^{n}\varepsilon_i\} \le nz.$$

Obviously, the optimal $z^\star$ is

$$\begin{aligned}
z^\star &= \frac{1}{n}\max\{n(5-r)-\sum_{i=1}^{n}\varepsilon_i, n(-5+r)+\sum_{i=1}^{n}\varepsilon_i\}\\
&= \frac{1}{n}|n(5-r)-\sum_{i=1}^{n}\varepsilon_i| = \frac{1}{n}\left(n(5-r)-\sum_{i=1}^{n}\varepsilon_i\right)\\
&\ge \frac{1}{n}n(5-r) = |5-r| = \psi(0).
\end{aligned}$$

In order to prove assertion (ii), let us suppose that $a^{(i)}, a^{(j)} \in \partial K_r^{(2)}$ are arbitrary. Then there holds

$$\begin{aligned}
d_2(a^{(i)}, a^\star) = d_2(a^{(j)}, a^\star) &\Leftrightarrow \|a^{(i)}-a^\star\|_2^2 = \|a^{(j)}-a^\star\|_2^2\\
&\Leftrightarrow \left(a^{(j)}-a^{(i)}\right)(a^\star)^T = \frac{1}{2}\left(\|a^{(j)}\|^2-\|a^{(i)}\|^2\right)\\
&\Leftrightarrow 5\left(\sum_{s=1}^{n} r_s^{(j)} - \sum_{s=1}^{n} r_s^{(i)}\right) = \frac{1}{2}\left(\|a^{(j)}\|^2-\|a^{(i)}\|^2\right)\\
&\Leftrightarrow 5\left(d_1(a^{(j)}, a^\star)-d_1(a^{(i)}, a^\star)\right) = \frac{1}{2}\left(\|a^{(i)}\|_2^2-\|a^{(j)}\|_2^2\right). \qquad (21)
\end{aligned}$$

From (21) it can be seen that $\|a^{(i)}\|_2 \ge \|a^{(j)}\|_2$, if and only if $d_1(a^{(j)}, a^\star) \ge d_1(a^{(i)}, a^\star)$, from where there follow (13) and (14). □

Remark 2. A generalization of results from Theorem 1 and Theorem 2 could be written for an arbitrary d_p ($p \ge 1$) distance, but the proof would require us to solve a nondifferentiable optimization problem (see e.g. Avriel (2003); Ruszczynski (2006)). It should also be noted that the cases d_1, d_2, d_∞ are quite sufficient for the applications in question.

4 Ranking internal research projects at the University of Osijek

As an illustration, we consider the problem of ranking projects of the internal research program at the University of Osijek (INGI-2015) described in Example 1. We will analyze only the problem of ranking $m = 10$ Social Sciences and Humanities project proposals. The AWGs from two independent reviewers are shown in Table 4 and in Fig. 4.

The Committee for Research Project Evaluation decided to apply the Euclidean d_2-distance with corrections by the Chebyshev d_∞-distance in terms of Theorem 1 and Theorem 2, i.e., if two projects are approximately equally d_2-ranked, then we give priority to the project with more uniform evaluation grades, i.e. to the project that is d_∞-ranked better.

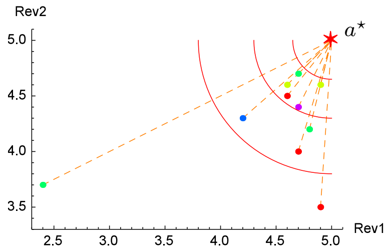

Figure 4: Ranks of $m = 10$ Social Sciences and Humanities projects

Note that projects `INGI-1` and `INGI-2` are equally d_2-ranked, but the project `INGI-1` is d_∞-ranked better (has more uniform evaluation grades), hence it ranks first. If we tried to differentiate projects `INGI-1` and `INGI-2` by using more decimals in the d_2-rank, then the project `INGI-2` would be placed before the project `INGI-1`. A similar situation takes place with projects `INGI-3` and `INGI-4`.

No.	Code	`Rev#1`	`Rev#2`	d_2-distance	d_∞-distance
1	INGI-1	4.7	4.7	0.4	0.3
2	INGI-2	4.9	4.6	0.4	0.4
3	INGI-3	4.6	4.6	0.6	0.4
4	INGI-4	4.6	4.5	0.6	0.5
5	INGI-5	4.7	4.4	0.7	0.6
6	INGI-6	4.8	4.2	0.8	0.8
7	INGI-7	4.7	4.0	1.0	1.0
8	INGI-8	4.2	4.3	1.1	0.8
9	INGI-9	4.9	3.5	1.5	1.5
10	INGI-10	2.4	3.7	2.9	2.6

Table 4: Ranking of Social Sciences and Humanities projects

5 Conclusions

Project ranking is a sensitive issue in multi-criteria decision making. During the evaluation process, it can be expected that two or more projects are roughly equally ranked in relation to the selected distance function. We believe that it is not appropriate to rank such projects by using more decimal places, but that the project with more uniform evaluation grades should be positioned better. The paper shows how this can be achieved by combining different distance functions.

The presented method can be applied to other different situations like department ranking inside a university, ranking teachers and associates on the basis of a university survey or on the basis of the quality of scientific research, ranking administrative staff on the basis of a survey, etc.

Acknowledgements.

This work is supported by the Ministry of Science, Education and Sports, Republic of Croatia. The authors would also like to thank the administration of Josip Juraj Strossmayer University of Osijek for preparing the data for this paper, and Mrs. Ivanka Ferčec for significantly improving the use of English in the paper.

References

Avriel, M., 2003. Nonlinear Programming: Analysis and Methods. Dover Publications, Inc. Mineola, New York.

Cocchi, D., Cavaliere, G., Freo, M., Giannerini, S., Mazzocchi, M., Trivisano, C., Viroli, C., 2014. A support for classifying scientific papers in a university department. Procedia Economics and Finance 17, 47–54.

Collan, M., Fedrizzi, M., Luukka, P., 2013. A multi-expert system for ranking patents: An approach based on fuzzy pay-off distributions and a TOPSIS-AHP framework. Expert Systems with Applications 40, 4749–4759.

Daraio, C., Bonaccorsi, A., Simar, L., 2015. Rankings and university performance: a conditional multidimensional approach. European Journal of Operational Research Doi: 10.1016/j.ejor.2015.02.005.

Kadziński, M., Słowiński, R., 2015. Parametric evaluation of research units with respect to reference profiles. Decision Support Systems 72, 33–43.

Karami, A., Johansson, R., 2014. Choosing dbscan parameters automatically using differential evolution. International Journal of Computer Applications 91, 1–11.

Málek, J., Hudečková, V., Matějka, M., 2014. System of evaluation of research institutions in the Czech Republic. Procedia Computer Science 33, 315–320.

Mandic, D., Jovanovic, P., Bugarinovic, M., 2014. Two-phase model for multi-criteria project ranking: Serbian railways case study. Transport Policy 36, 88–104.

Manouselis, N., Verbert, K., 2013. Layered evaluation of multi-criteria collaborative filtering for scientific paper recommendation. Procedia Computer Science 18, 1189 –1197.

Mardani, A., Jusoh, A., Zavadskas, E.K., 2015. Fuzzy multiple criteria decision-making techniques and applications - Two decades review from 1994 to 2014. Expert Systems with Applications 42, 4126–4148.

Morales-Esteban, A., Martínez-Álvarez, F., Scitovski, S., Scitovski, R., 2014. A fast partitioning algorithm using adaptive Mahalanobis clustering with application to seismic zoning. Computers & Geosciences 73, 132–141.

Rad, A., Naderi, B., Soltani, M., 2011. Clustering and ranking university majors using data mining and ahp algorithms: A case study in Iran. Expert Systems with Applications 38, 755–763.

Ruszczynski, A., 2006. Nonlinear Optimization. Princeton University Press, Princeton and Oxford.

Saaty, R.W., 1990. How to make a decision: The analytic hierarchy process. European Journal of Operational Research 18, 9–26.

Saaty, T.L., 1980. The Analytic Hierarchy Process. Mc-Graw Hill.

Ž. Turkalj, Markulak, D., Singer, S., Scitovski, R., 2016. Research project grouping and ranking by using adaptive Mahalanobis clustering. Croatian Operational Research Review 7, 81–96.